Living up to expectations: estimating direct and indirect rebound effects for UK households

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Keywords
rebound effects, econometric models, economic analysis, energy efficiency measures

Abstract
This study estimates the combined direct and indirect rebound effects from various types of energy efficiency improvement by UK households. In contrast to most studies of this topic, we base our estimates on cross-price elasticities and therefore capture both the income and substitution effects of energy efficiency improvements. Our approach involves: a) estimating a household demand model to obtain price and expenditure elasticities of different goods and services; b) utilising a multiregional input-output model to estimate the GHG emission intensities of those goods and services; c) combining the two to estimate direct and indirect rebound effects; and d) decomposing those effects to reveal the relative contribution of different mechanisms and commodities. We estimate that the total rebound effects are 63 % for measures that improve the efficiency of domestic gas use, 53 % for electricity use and 46 % for vehicle fuel use. The primary source of this rebound is increased consumption of the cheaper energy service (i.e. direct rebound) and this is primarily driven by substitution effects. Our results suggest that the neglect of substitution effects may have led prior research to underestimate the total rebound effect. However, we provide a number of caveats to this conclusion, as well as indicating priorities for future research.

Introduction
'Rebound effects' is a widely used term for a variety of economic responses to improved energy efficiency. The net result of these effects is typically to increase energy consumption and greenhouse gas (GHG) emissions relative to a counterfactual baseline in which these responses do not occur. To the extent that rebound effects are neglected in policy appraisals, the energy and emissions 'saved' by such measures may be less than anticipated.

Studies of rebound effects for consumers typically focus upon the *direct* effects that result from increased consumption of cheaper energy services. For example, fuel-efficient cars make driving cheaper so people may drive further and/or more often [1, 2]. But a comprehensive accounting of the global environmental impact of energy efficiency improvements must also take into account various *indirect* rebound effects. For example, any savings on fuel bills may be put towards increased consumption of other goods and services whose provision also involves energy use and emissions at different stages of their global supply chains – such as laptops that have been made in China and shipped to the UK [3, 4]. To quantify indirect rebound effects, it is necessary to combine econometric analysis of household (re)spending patterns with estimates of the energy and emissions 'embodied' within different categories of goods and services. The latter, in turn can be derived from environmentally extended, multiregional input-output models [5–7].

Relatively few studies estimate *both* direct and indirect rebound effects and most of these rely upon expenditure elasticities rather than cross-price elasticities. As a result, they only capture the *income effects* of energy efficiency improvements and fail to capture the *substitution effects* [8]. *Income effects* are defined as the changes in consumption that result solely from the increase in real income provided by an energy efficiency improvement (cheaper energy services means that consumers can buy more goods), while *substitution effects* are defined as the changes in consumption that would result *if* real income were adjusted to keep consumer 'utility' constant. The observed change in consumption is the sum of the two. To better understand this the distinction, consider a household that installs insulation and recovers the capital costs over ten years through lower heating bills. Since the bill savings exactly offset the capital costs, there is no increase in real income over this period so the income effect is zero. Hence, studies that focus solely upon income effects would estimate the direct and indirect rebound effects over that period to be zero as well. But since the unit cost of heating has fallen relative to that of other goods and services, the household is likely to consume more heating and *fewer* goods and services that are 'substitutes' to heating. At the same time, the household may consume *more* of other goods and services that are 'complements' to heating.[1] The net result will be a shift in consumption patterns and hence a change in the GHG emissions associated with that consumption that may offset the original emission savings. Hence, it is possible that studies that neglect substitution effects will underestimate rebound effects.

This study therefore addresses the limitations of the existing literature by: a) estimating the magnitude of both direct and indirect rebound effects following the adoption of energy efficiency measures by households; b) identifying the relative contribution of income and substitution effects to these results; and c) identifying the relative contribution of individual goods and services. This is the first study to estimate these effects for UK households, as well as the first to decompose them to this level of detail.

The paper first introduces the relevant concepts, highlights some methodological trade-offs and summarises the existing literature. Then it outlines the methodology, including the data sources used, the economic model adopted and the econometric techniques employed. After that, the results are presented, including the estimates of direct and indirect rebound effects and the contribution of different mechanisms and commodities to those effects. The last sections discuss the robustness of the results and highlights some implications, and conclude the paper.

Concepts and previous work

DIRECT REBOUND EFFECTS

Cost-effective energy efficiency improvements reduce the effective price of energy services such as heating and lighting, thereby encouraging increased consumption of those services that offsets the initial energy and emission savings. The marginal change in the energy (q_e) required to provide a given quantity of energy service (q_s) following a marginal change in energy efficiency ($\varepsilon = q_s / q_e$) may be expressed as:

$$\eta_{q_e,\varepsilon} = \frac{\partial \ln q_e}{\partial \ln \varepsilon} \quad (1)$$

As shown by Sorrell and Dimitropoulos [9], this 'efficiency elasticity of energy demand' may be written as:

$$\eta_{q_e,\varepsilon} = -\eta_{q_s,p_s} - 1 \quad (2)$$

Where η_{q_s,p_s} is the elasticity of demand for the energy service (q_s) with respect to the energy cost of that service ($p_s = p_e / \varepsilon$). The negative of this elasticity is commonly taken as a measure of the *direct rebound effect*: $R_D = -\eta_{q_s,p_s}$ [9]. If the energy service is a normal good ($0 \geq \eta_{q_s,p_s}$) there will be a positive direct rebound effect ($R_D \geq 0$). This elasticity may be decomposed into a *substitution effect* and an *income effect* using the 'Slutsky equation':

$$\eta_{q_s,p_s} = \tilde{\eta}_{q_s,p_s} - w_s \eta_{q_s,x} \quad (3)$$

Where: w_s is the share of the energy service in total household expenditure (x); $\eta_{q_s,x}$ is the expenditure elasticity of the energy service ($\eta_{q_e,\varepsilon} = \delta \ln q_e / \delta \ln \varepsilon$); and $\tilde{\eta}_{q_s,p_s}$ is the *compensated* own-price elasticity of demand for the energy service, holding utility constant. The income ($\hat{R}_D$) and substitution ($\tilde{R}_D$) components of the direct rebound effect are then as follows:

$$\hat{R}_D = w_s \eta_{q_s,x} \qquad \tilde{R}_D = -\tilde{\eta}_{q_s,p_s} \qquad R_D = \hat{R}_D + \tilde{R}_D \quad (4)$$

Income and substitution effects may either offset or reinforce one another. If estimates of η_{q_s,p_s} are available the 'full' direct rebound effect can be derived, and if estimates of $\eta_{q_s,x}$ are also available it can be decomposed into income and substitution effects. In contrast, if only estimates of $\eta_{q_s,x}$ are available, then only the income effect can be obtained. This will form a biased estimate of the direct rebound effect since substitution effects will be overlooked.

INDIRECT REBOUND EFFECTS

Energy efficiency improvements may also change the quantity demanded of other goods and services. These include both other energy services (e.g. heating) and other non-energy goods and services (e.g. furniture) that 'embody' the energy and emissions required to manufacture, transport and deliver them. These changes in consumption patterns will impact energy use and emissions at each stage of the relevant supply chains. From a global perspective, these changes may either offset or add to the energy and emission savings from the energy efficiency improvement depending on whether the quantity demanded of the relevant goods or service has increased or fallen. The *indirect rebound effect* (R_{I_i}) from an individual commodity (i) will be proportional to this change in energy and emissions, which in turn will depend upon *both* the energy or emissions intensity of the commodity relative to that of the energy service *and* the elasticity of demand for the commodity with respect to the price of the energy service. The latter is defined as:

$$\eta_{q_i,p_s} = \frac{\partial \ln q_i}{\partial \ln p_s} \quad (5)$$

1. The cross price elasticity between gas and a substitute good is positive, while that between gas and a complementary good is negative.

Again, this elasticity can be decomposed into income and substitution effects using the Slutsky equation:

$$\eta_{q_i,p_s} = \tilde{\eta}_{q_i,p_s} - w_s \eta_{q_i,x} \quad (6)$$

Where: w_s is the share of the energy service in total household expenditure; $\eta_{q_i,x}$ is the expenditure elasticity of commodity *i*; and $\tilde{\eta}_{q_i,p_s}$ is the compensated elasticity of demand for commodity *i* with respect to the energy cost of the energy service. The substitution effect for commodity *i* ($\tilde{\eta}_{q_i,p_s}$) may offset or reinforce the income effect ($-w_s\eta_{q_i,x}$) for that commodity. Consumption of commodities that are complements (substitutes) to the energy service will increase (reduce) following the energy efficiency improvement. The impact of this on emissions will depend upon the emissions intensity of each commodity. If estimates of both η_{q_i,p_s} and $\eta_{q_i,x}$ are available the full indirect rebound effect for each commodity (*i*) can be derived and decomposed into income and substitution effects ($R_{I_i} = \hat{R}_{I_i} + \tilde{R}_{I_i}$). But if only estimates of $\eta_{q_i,x}$ are available, only the income effect can be obtained. To estimate the overall indirect rebound effect we need to sum the corresponding change in emissions over all commodities ($R_I = \sum_i R_{I_i}$).

ESTIMATING DIRECT AND INDIRECT REBOUND EFFECTS

To estimate direct and indirect rebound effects we need the own- and cross-price elasticities for the relevant energy service. This requires the estimation of a *household demand model* – namely, a system of *n* equations representing household demand for *n* commodities as a function of total expenditure, commodity prices and other variables, with one of these commodities being the energy service (*s*).

A growing number of studies estimate own-price elasticities for individual energy services (η_{q_s,p_s}), but to our knowledge no study has estimated cross price elasticities (η_{q_j,p_s}) owing the difficulties of specifying energy services as a 'commodity' within a household demand model [10]. Since energy services are produced from a combination of energy commodities (e.g. gas) and durable goods (e.g. boilers), specifying their energy cost ($p_s = p_e / \varepsilon$) and quantity demanded (q_s) involves combining data on energy commodity purchases with additional data on the ownership and energy efficiency of the relevant durables [11]. Since this data may not be available, a simpler alternative is to estimate a model for purchased commodities (*i*) and to simulate energy efficiency improvements by a reduction in the price of the relevant energy commodities (*e*) [e.g. 12]. So, for example, more efficient boilers may be simulated by a reduction in the unit price of natural gas (p_e), since both will reduce the energy cost of heating (p_s). Elasticities may then be estimated with respect to energy commodity prices (p_e), rather than energy service prices (p_s) and used to estimate both direct and indirect rebound effects. This approach is simpler to implement but may lead to biased estimates of rebound effects (see below).

It is common to formulate household demand models in terms of expenditures (x_i) rather than quantity demanded (q_i) since expenditures are easier to measure. The following relationships may be derived:

$$\eta_{q_i,p_i} = \eta_{x_i,p_i} - 1; \quad \tilde{\eta}_{q_i,p_j} = \tilde{\eta}_{x_i,p_i} - 1; \quad \eta_{q_i,p_j} = \eta_{x_i,p_j};$$

$$\tilde{\eta}_{q_i,p_j} = \tilde{\eta}_{x_i,p_j}; \quad \eta_{q_i,x} = \eta_{x_i,x} \quad (7)$$

Household demand models can be estimated from pooled cross-sectional data on household expenditures and commodity prices. But the number of coefficients to be estimated limits the degrees of freedom, with the result that expenditures need to be aggregated into a limited number of commodity groups. For the same reason, such models provide limited scope for including covariates and typically require restrictions to be imposed upon the parameter values to increase the degrees of freedom. A common strategy is to assume *separability* of preferences between aggregate commodity groups such as food and transport, implying that decisions on how much to spend on one group (e.g. transport) are separate from decisions on how to allocate this expenditure between the goods and services within that group (e.g. bus, car or train travel) [13].[2] This is a restrictive assumption, but it can work reasonably well if the categories are well chosen

Methodology

Our approach involves estimating a household demand model to derive price and expenditure elasticities of different goods and services, utilising a multiregional input-output model to estimate the GHG emission intensities of those goods and services, combining the two to estimate direct and indirect rebound effects, and decomposing those effects to reveal the relative contribution of different mechanisms and commodities. The first section develops expressions for these effects, the second describes the econometric model and the third summarises the data.

REBOUND MODEL

Assume a household makes a costless investment that increases the energy efficiency (ε) of providing an energy service (*s*) by $\varsigma = \Delta\varepsilon / \varepsilon$ ($\varsigma \geq 0$), thereby reducing the energy cost (p_s) of that service by $\tau = \Delta p_s / p_s$ ($\tau \leq 0$). Let Q represent the household's baseline GHG emissions (direct plus embodied), ΔH the change in emissions that would occur *without* any behavioural responses to the lower cost energy service (the 'engineering effect'), ΔG the change in emissions that results from those behavioural responses (the 're-spending effect'), and $\Delta Q = \Delta H + \Delta G$ the net change in GHG emissions. The total rebound effect (R_T) is then given by:

$$R_T = (\Delta H - \Delta Q) / \Delta H = -\Delta G / \Delta H \quad (8)$$

As discussed above, this is comprised of direct and indirect rebound effects ($R_T = R_D + R_I$) which may each be decomposed into income and substitution effects ($R_D = \hat{R}_D + \tilde{R}_D$ and $R_I = \hat{R}_I + \tilde{R}_I$). The baseline GHG emissions for the household may be written as:

$$Q = x_s u_s^x + \sum_{i(i \neq s)} u_i^x x_i \quad (9)$$

2. 'Weak separability' implies that the marginal rate of substitution between commodities in one group is independent of the quantities of other commodities in other groups. This allows the demand for commodities within a group to be written solely as a function of the expenditure on the group and the prices of commodities within the group, with the prices of other commodities only affecting the group expenditure and not the allocation of expenditure within the group.

Table 1. Formulae for the components of the rebound effect.

	Direct rebound effect	Indirect rebound effect for commodity *i*
Income effect	$\hat{R}_D = w_e \eta_{q_e,x}$	$\hat{R}_{I_i} = \psi_i w_e \eta_{q_i,x}$
Substitution effect	$\tilde{R}_D = -\tilde{\eta}_{q_e,p_e}$	$\tilde{R}_{I_i} = -\psi_i \tilde{\eta}_{q_i,p_e}$

Where x_i is the expenditure on commodity *i* (in £), u_i^x is the GHG intensity of that expenditure (in tCO_{2e}/£), and x_s and u_s^x are the corresponding values of these variables for the energy service. The GHG intensities include both direct and embodied emissions. To estimate the engineering effect (ΔH), we assume the consumption of all commodities remains unchanged while the energy cost of the energy service falls. The change in expenditure on the energy service as a consequence of the engineering effect is then given by $\Delta x_s^H = q_s \Delta p_s$. Given that $\Delta p_s = \tau p_s$ and $\Delta H = u_s^x \Delta x_s^H$ we obtain the following expression:

$$\Delta H = u_s^x x_s \tau \quad (10)$$

To estimate the re-spending effect (ΔG), we must allow for the change in expenditure on each commodity group (Δx_i).The change in expenditure on the energy service itself as a consequence of the engineering effect is given by $\Delta x_s^G = p_s \Delta q_s$.[3] Adding in the change of expenditure on other commodity groups we obtain the following expression:

$$\Delta G = u_s^x \Delta x_s^G + \sum_{i(i \neq s)} u_i^x \Delta x_i \quad (11)$$

Assuming marginal changes, we can use elasticities to substitute for Δx_s^G and Δx_i in this equation:

$$\Delta G = u_s^x x_s \tau (\eta_{x_s,p_s} - 1) + \sum_{i(i \neq s)} u_i^x x_i \tau \eta_{x_i,p_s} \quad (12)$$

Substituting the expressions for ΔH (Equation 10) and ΔG (Equation 12) into Equation 8 and noting that $w_i = x_i / x$, we arrive at the following expression for the total (direct plus indirect) rebound effect:

$$R_T = (1 - \eta_{x_s,p_s}) - \sum_{i(i \neq s)} \psi_i \eta_{x_i,p_s} \quad (13)$$

Where:

$$\psi_i = (u_i^x w_i)/(u_s^x w_s) \quad (14)$$

Using Equation 7, the total rebound effect can also be expressed as:

$$R_T = -\eta_{q_s,p_s} - \sum_{i(i \neq s)} \psi_i \eta_{q_i,p_s} \quad (15)$$

The first term in Equation 15 is the direct rebound effect (R_D) and the second is the indirect effect (R_I). The first depends solely upon the own-price elasticity of energy service demand (η_{q_s,p_s}), while the second depends upon *both* the elasticity of demand for commodity *i* with respect to the energy service (η_{q_i,p_s}) *and* the GHG intensity and expenditure share of that commodity relative to that of the energy service (ψ_i). Hence, commodities with a small cross price elasticity may nevertheless contribute a large indirect rebound effect if they are relatively GHG intensive and/or have a large expenditure share (and vice versa). Using the Slutsky equation, we decompose Equation 15 as follows:

$$R_T = \left[w_s \eta_{q_s,x} - \tilde{\eta}_{q_s,p_s} \right] + \left[\sum_{i(i \neq s)} \left[\psi_i w_s \eta_{q_i,x} - \psi_i \tilde{\eta}_{q_i,p_s} \right] \right] \quad (16)$$

As noted, the challenges of incorporating energy services within a household demand model make it difficult to implement this approach directly. Hence, in what follows we estimate the required elasticities with respect to energy commodities (*e*) rather than energy services (*s*). Table 1 summarises the required expressions.

ECONOMETRIC MODEL

We base our household demand model on the *Linear Approximation to the Almost Ideal Demand System* (*LAIDS*) which has a number of advantages over competing approaches [14]. As a compromise between resolution and degrees of freedom, we split household expenditure into 12 subcategories (Table 2) and assume separability to give a two-stage budgeting framework. Households are assumed to first allocate expenditure between four aggregate groups (*r*), and then distribute the group expenditures between individual commodities within each group (*i*). This framework allows: a) expenditure on each group be specified as a function of the prices of the aggregate groups (stage 1); and b) expenditure on commodities within that group to be specified as a function of group expenditure and the prices of other commodities within that group (stage 2).

Let x_t^r represent the expenditure on aggregate commodity group *r* in period *t* and w_t^r the fractional share of that group in total household expenditure ($w_t^r = x_t^r / x_t$). In the first stage of the LAIDS model, this is specified as:

$$w_t^r = \alpha^r + \sum_{s=1,..4} \gamma^{rs} \ln p_t^s + \beta^r \ln(x_t / P_t) + \sum_{s=1,..3} \lambda^{rs} w_{t-1}^s + \varepsilon_t^r \quad (17)$$

Where: *r* and *s* index over the aggregate commodity groups; p_t^s is the price of the aggregate commodity group *s* in period *t*; x_t is total expenditure per household in that period; P_t is the Stone's price index for the aggregate commodities; w_{t-1}^s is the lagged expenditure share of commodity group *s*; α^r, γ^{rs}, β^r and λ^{rs} are the unknown parameters and e_t^r is the error term. The Stone's price index is defined as:

$$\ln P_t = \sum_{r=1,..4} w_t^r \ln p_t^r \quad (18)$$

3. For the energy service itself, the total change in expenditure is the sum of the engineering and re-spending effects: $\Delta x_s = \Delta x_s^H + \Delta x_s^G$

Table 2. Categories of goods and services.

Aggregate Group (r) Stage 1	Category (i) Stage 2	COICOP category	Description
Food and beverages	1	1	Food and non-alcoholic beverages
	2	2	Alcoholic beverages, tobacco, narcotics
Transport	3	7.2.2.2	Vehicle fuels and lubricants
	4	Rest of 7	Other transport
Energy	5	4.5.1	Electricity
	6	4.5.2	Gas
	7	4.5.3 and 4.5.4	Other fuels
Other goods and services	8	9	Recreation & culture
	9	11	Restaurants & hotels
	10	10	Education
	11	8	Communication
	12		
		3	Other
		4.1 to 4.4	Clothing and footwear
		5	Other housing
			Furnishings, household equipment & household maintenance
		6	Health
		12	Miscellaneous goods and services

Notes: COICOP – Classification of Individual Consumption According to Purpose. 'Other housing' includes rent, mortgage payments, maintenance, repair and water supply. 'Other transport' includes public transport, non-fuel expenditure on private vehicles and some aviation – although air travel for package holidays is included within 'recreation and culture'. 'Other fuels' include solids and liquids.

Given the constraints on degrees of freedom, we do not include additional covariates. However, our model departs from standard applications of LAIDS by including lagged expenditure shares (w^s_{t-1}) to capture the inertia in price responses – for example as a result of habit formation. The inclusion of lags also reduces problems of serial correlation [15–17]. Since the lagged expenditure shares sum to unity, we only include three in each equation to avoid multi-collinearity.

Restrictions are often imposed upon the parameter values to ensure the results are compatible with consumer demand theory. Specifically, *adding up* requires that expenditures on each commodity add up to total expenditure; *homogeneity* requires that quantity demanded remains unchanged if prices and total expenditure change by an equal proportion; and *symmetry* requires that the compensated cross-price elasticities between two commodities are equal. These restrictions may be implemented as follows:

Adding up: $\sum_r \alpha^r = 1; \quad \sum_r \beta^r = 0; \quad \sum_r \gamma^{rs} = 0 \quad s=1,..4;$

and $\sum_r \lambda^{rs} = 0 \quad s=1,..3;$

Homogeneity: $\sum_s \gamma^{rs} = 0 \quad s=1,..4;$

Symmetry: $\gamma^{rs} = \gamma^{sr}$

The second stage of the LAIDS model distributes the group expenditures (x^r_t) between individual commodities within each group. Let x^r_{it} represent expenditure on commodity i in aggregate group r during period t ($i \in r$) and w^r_{it} represent the fractional share of that commodity in the expenditure on group r ($w^r_{it} = x^r_{it} / x^r_t$).

The latter is specified as:

$$w^r_{it} = \alpha^r_i + \sum_{j=1,..k^r} \gamma^r_{ij} \ln p^r_{jt} + \beta^r_i \ln(x^r_t / P^r_t)$$

$$+ \sum_{j=1,..(k^r-1)} \lambda^r_{ij} w^r_{jt-1} + \varepsilon^r_{it} \qquad (19)$$

Where: i and j index over the commodities within aggregate group r ($i, j \in r$); k^r is the number of commodities in aggregate group r; p^r_{jt} is the price of commodity j in period t; x^r_t is expenditure on group r in that period; P^r_t is the Stone's price index for group r; α^r_i, γ^r_{ij}, β^r_i and λ^r_{ij} are the unknown parameters and ε^r_{it} is the error term. The Stone's price index for group r is defined as:

$$\ln P^r_t = \sum_{i=1,..k^r} w^r_{it} \ln p^r_{it} \qquad (20)$$

Again, the adding up, symmetry and homogeneity restrictions can be imposed as follows:

Adding up: $\sum_i \alpha^r_i = 1; \quad \sum_i \beta^r_i = 0; \quad \sum_i \gamma^{ij} = 0; \quad j = 1,..k^r$

and $\sum_i \lambda^r_{ij} = 0 \quad j = 1,.(k^r -1)$

Homogeneity: $\sum_i \gamma^r_{ij} = 0 \quad j = 1,..k^r$

Symmetry: $\gamma^r_{ij} = \gamma^r_{ji}$

Alternatively, an unrestricted model can be estimated for both first and second stage and the homogeneity and symmetry restrictions tested. It is common for these restrictions to be rejected in empirical studies [18]. The adding up restriction, however, is always satisfied by dropping one of the equations.

Box 1. Interpretation of the between-group, within-group and total elasticities.

1. *Between-group* expenditure ($\eta_{x_r,x}$) and price (η_{x_r,p_s} and $\tilde{\eta}_{x_r,p_s}$) elasticities for the aggregate commodity groups (r) respectively indicate how expenditure on aggregate group r changes following: a) a change in total expenditure; and b) a change in the price of aggregate group s holding total expenditure fixed.
2. *Within-group* expenditure ($\eta^r_{x_i,x_r}$) and price ($\eta^r_{x_i,p_j}$ and $\tilde{\eta}^r_{x_i,p_j}$) elasticities for each commodity i within aggregate group r respectively indicate how expenditure on this commodity changes following: a) a change in expenditure on group r; and b) a change in the price of commodity j within aggregate group r holding expenditure on group r fixed. Here, both i and j are within the same aggregate group.
3. *Total* expenditure ($\eta_{x_i,x}$) and price (η_{x_i,p_j} and $\tilde{\eta}_{x_i,p_j}$) elasticities for each commodity i within aggregate group r respectively indicate how expenditure on this commodity changes following: a) a change in total expenditure; and b) a change in the price of commodity j holding total expenditure fixed but allowing expenditure on group r to vary. Here, i and j may be within the same or different aggregate group.

Godard [19] derives equations for estimating the short run expenditure and price elasticities for a single stage LAIDS model, while Edgerton [17] derives expressions for a two-stage model. In the latter, 'total' elasticities are calculated from estimates of the 'between-group' and 'within-group' elasticities. The interpretation of these is summarised in Box 1 while the relevant formulae are summarised in Table 3 [17]. Here, δ_{rs} (Kronecker delta) is equal to unity when $r=s$ (i.e. own-price elasticity) and zero otherwise. Similarly, δ^r_{ij} is unity when $i=j$ and zero otherwise.

The formula for the total expenditure elasticity for the i^{th} commodity in the r^{th} group (Table 3, line 2) is simply the product of the within-group elasticity for that commodity and the expenditure elasticity of the group.

The formula for the total uncompensated price elasticity (Table 3, line 3) is more complex. Note first that when commodities i and j are in different groups, $\delta_{rs} = 0$ and the expression reduces to:

$$\eta_{x_i,p_j} = \eta^r_{x_i,x_r}\eta_{x_r,p_s}w^s_j \tag{21}$$

Here, the first term ($\eta^r_{x_i,x_r}$) represents the change in expenditure on commodity i following a change in expenditure on group r; the second term represents the change in expenditure on group r following a change in the price of group s; and the third term represents the share of commodity j in the expenditure on group s. As shown by Edgerton [17], the latter is equivalent to the change in the price of group s following a change in the price of commodity j ($w^s_j = \partial \ln p_s / \partial \ln p_j$). When i and j are in the same group ($r=s$), the expression becomes:

$$\eta_{x_i,p_j} = \eta^r_{x_i,p_j} + \eta^r_{x_i,x_r}(1+\eta_{x_r,p_r})w^r_j \tag{22}$$

Here, the total cross price elasticity equals the within-group cross price elasticity ($\eta^r_{x_i,p_j}$), plus a product of three terms. The first of these ($\eta^r_{x_i,x_r}$) measures the change in expenditure on commodity i following a change in expenditure on group r; the second measures the change in expenditure on group r following a change in the price of group r; and the third represents the change in the price of group r following a change in the price of commodity j ($w^r_j = \partial \ln p_r / \partial \ln p_j$). The smaller each of these terms are, the smaller the difference between the within-group and total price elasticity. The formula for the total compensated price elasticity (Table 3, line 4) follows a similar pattern. Following standard practice, we estimate the elasticities using the mean values of the expenditure shares over the full time series. The total elasticities are used for estimating rebound effects.

DATA

Data for the price of different commodity groups and household expenditure on those groups is taken from *Consumer Trends*, published by the UK Office of National Statistics (ONS). The period chosen is 1964 to 2013 and the values are converted to current prices using a base year of 2010. Data on total household numbers for selected years is taken from DGLC [20], with data on intermediate years estimated by linear interpolation. During this period, the share of food in total expenditure almost halved, the share of transport increased by 50 % and the

Table 3. Analytical expressions for elasticities within a two-stage LAIDS model.

Elasticity	Between-group	Within-group ($i, j \in r$)	Total
Expenditure	$\eta_{x_r,x} = 1 + \dfrac{\beta^r}{w^r}$	$\eta^r_{x_i,x_r} = 1 + \dfrac{\beta^r_i}{w^r_i}$	$\eta_{x_i,x} = \eta^r_{x_i,x_r}\eta_{x_r,x}$
Uncompensated price	$\eta_{x_r,p_s} = \dfrac{\gamma^{rs} - \beta^r w_s}{w_r} - \delta_{rs}$	$\eta^r_{x_i,p_j} = \dfrac{\gamma^r_{ij} - \beta^r_i w^r_j}{w^r_i} - \delta^r_{ij}$	$\eta_{x_i,p_j} = \delta_{rs}\eta^r_{x_i,p_j} + \eta^r_{x_i,x_r}(\delta_{rs} + \eta_{x_r,p_s})w^s_j$
Compensated price	$\tilde{\eta}_{x_r,p_s} = \dfrac{\gamma^{rs}}{w_r} + w_s - \delta_{rs}$	$\tilde{\eta}^r_{x_i,p_j} = \dfrac{\gamma^r_{ij}}{w^r_i} + w^r_j - \delta^r_{ij}$	$\tilde{\eta}_{x_i,p_j} = \delta_{rs}\tilde{\eta}^r_{x_i,p_j} + \eta^r_{x_i,x_r}(\delta_{rs} + \tilde{\eta}_{x_r,p_s})w^s_j$

share of energy fell by 30 %. Within the energy group, substitution by gas reduced the expenditure share of other fuels (coal and oil) from 42 % in 1964 to 6 % in 2013.

Our data source for the GHG emissions associated with different categories of goods and services is the *Surrey Environmental Lifestyle Mapping Framework* (SELMA). This is a quasi-multi-regional, environmentally extended input-output model that provides estimates of the GHG intensity of UK household expenditure in each category (in tCO_2e/£) for 2004 [21].[4] These figures include both the *direct* emissions from the consumption of electricity, heating fuels and vehicle fuels, and the *embodied* emissions from each stage of the supply chain for goods and services – which may occur either in the UK or overseas. We adjust these estimates to allow for the emissions associated with government expenditure of product taxation revenues [8].

Figure 1 (top) shows that expenditure on electricity, gas and other fuels is approximately twice as GHG intensive as expenditure on vehicle fuels and approximately four times as GHGs intensive as expenditure on other transport – which is the next most GHG intensive category. Overall, expenditure on energy commodities is approximately five times as GHG intensive as the share-weighted mean. But the high GHG intensity of energy commodities is offset by their small share of total expenditure (7 % – Figure 1, middle), with the result that direct energy consumption only accounts for 27 % of an average household's 'GHG footprint' (Figure 1, bottom), split between 19 % domestic energy (i.e. electricity, gas and other fuels) and 8 % vehicle fuels.

The category providing the largest single contribution (25 %) to total emissions is 'other goods and services' which includes expenditure on clothing, housing maintenance, water and furnishings and accounts for 45 % of expenditure. The next highest is 'other transport' (12 %) which includes non-fuel costs for private cars, public transport and some air travel. Since these categories have both a relatively high expenditure share and a relatively high GHG intensity they provide a significant contribution to total emissions (42 %).

Our estimates of GHG intensities allow for the variation of product taxation between categories: namely VAT exemption for food and non-alcoholic beverages, lower rate VAT for domestic energy and high taxation of vehicle fuels (~60 % of retail price) [8]. The latter contributes to the comparatively low GHG intensity of vehicle fuels compared to domestic energy.

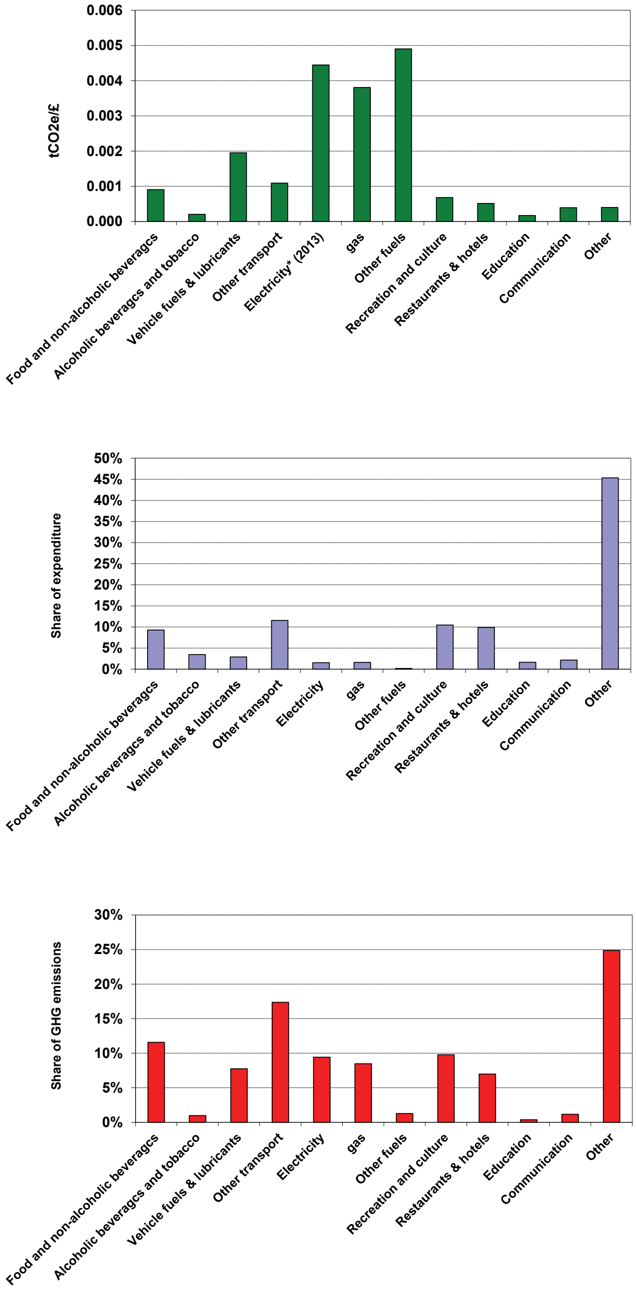

Figure 1. GHG intensity of expenditure, share of total expenditure and share of total GHG emissions by category for an average household.

Results

ECONOMETRIC ESTIMATES

Our two-stage model leads to a total of 16 equations in five groups. The equations in each group are estimated as a system using the Iterative Seemingly Unrelated Regressions (ISUR) method which is suitable for imposing cross-equation restrictions and corrects the estimates for any correlation of the error terms between equations. The adding up restriction is imposed by dropping one of the equations in each group. The equations are first estimated without imposing homogeneity and symmetry restrictions. A Wald test is then used to test for these restrictions both individually and in combination. If homogeneity and/or symmetry are not rejected then they are imposed upon the relevant group.

Full results for the parameter estimates and restriction tests are available from the authors. The overall fit of the equations is good, with more than two thirds of the parameter estimates being statistically significant at the 5 % level and with all but one of the equations having an adjusted R^2 exceeding 90 %. Both homogeneity and symmetry are rejected for the energy group, so we use the non-restricted results for this group. For all other

4. The GHG intensity of a category is estimated from the GHG emissions associated with that category in 2004 (obtained from SELMA) divided by 'real' expenditure on that category in 2004 (reference year 2013). The exception is electricity where emissions are estimated from 2012 electricity consumption (in kWh) multiplied by an emission factor for 2012 ($kgCO_2e$/kWh).

groups only the homogeneity restriction cannot be rejected. Hence, we impose homogeneity in all non-energy groups, but we do not impose symmetry on any group. Using the Portmanteau test, we find no evidence of serial correlation.

The most relevant results are the total elasticity estimates for the energy and transport groups which are summarised in Tables 8 to 10. For ease of interpretation, all elasticities are expressed for quantities (q) rather than expenditure shares (w). We make several observations. First, the expenditure elasticities for domestic energy are relatively low: 0.07 for electricity and 0.15 for gas. These values are broadly comparable with those estimated from cross-sectional data in our previous work [8] where we showed that high-income groups have very low expenditure elasticities for these commodities – which in turn has a disproportionate influence on the mean for all households. In contrast, the estimated expenditure elasticities for vehicle fuels, 'other transport' and the sub-categories of 'other goods and services' all exceed unity, indicating that they are luxury goods.

Second, the own-price elasticities for energy commodities have the expected sign with values of -0.39 for electricity, -0.59 for gas and -0.59 for vehicle fuels. For comparison, a review of studies by Espey and Espey [22] found a mean short-run elasticity of -0.35 (median -0.28) for electricity; a study by Asche *et al* [23] found short run elasticities of household natural gas demand to be -0.25 or less; and a review of studies by Goodwin*, et al.* [24] found a mean short-run elasticity for vehicle fuels of -0.25. Hence, our estimates appear to be at the high end of the range found in the literature – especially for gas and vehicle fuels. Since the expenditure elasticities for these commodities are relatively small, the own-price response is primarily driven by substitution effects – as is indicated by the near equivalence of the compensated and uncompensated elasticities for these commodities (Tables 9 and 10).

Third, electricity and gas are found to be substitutes, and both of these are estimated to be substitutes for vehicle fuels when the price of the latter changes, but complements when their own-price changes (symmetry is not imposed). In addition, 'other transport' and all subcategories within 'other goods and services' are estimated to be complements to energy commodities and therefore contribute a negative indirect rebound effect. In contrast, food and drink products are estimated to be substitutes and contribute a (small) positive indirect rebound effect.

Overall, the results suggest that the substitution effects for energy commodities outweigh the income effects, and changes in the price of one or more energy commodities have their largest impact on the quantity of energy commodities demanded. Since energy commodities are more GHG intensive, they will dominate the total rebound effect. This is demonstrated below, where we report the rebound results.

ESTIMATES OF REBOUND EFFECTS

The estimated rebound effects are presented in three ways to illustrate both their magnitude and their underlying drivers. Specifically, we indicate the relative contribution of: a) income and substitution effects; b) direct and indirect rebound effects; and c) direct and embodied emissions.

Our estimates of the *total* rebound effect are 63 % for gas, 53 % for electricity and 46 % for vehicle fuels, (Figure 2). These estimates are larger than most in the literature, although smaller than those by Brannlund *et al* [12] and Mizobuchi [25] who use a similar methodology. Net substitution across all commodities accounts for between two thirds and three quarters of the total rebound for electricity and gas, but only one fifth for vehicle fuels. This demonstrates the importance of capturing substitution effects and suggests that studies that only estimate income effects could underestimate the total rebound – particularly for electricity and gas.

Our estimates of *direct* rebound effects are 58 % for gas, 41 % for electricity and 59 % for vehicle fuels, indicating that they account for the majority of the total rebound. For vehicle fuels, the direct rebound effect *exceeds* the total rebound effect, since the indirect rebound effect is negative. These estimates are higher than most in the literature, particularly for vehicle fuels where the majority of studies estimate direct rebound effects of 20 % or less [26]. Figure 3 demonstrates that the contribution of income effects to the total rebound derives from other commodities (indirect rebound) while the contribution of substitution effects mostly derives from energy itself (direct rebound). Again, studies that only estimate income effects could erroneously conclude that the indirect rebound effect accounts for the majority of the total rebound – whereas these results show the opposite.

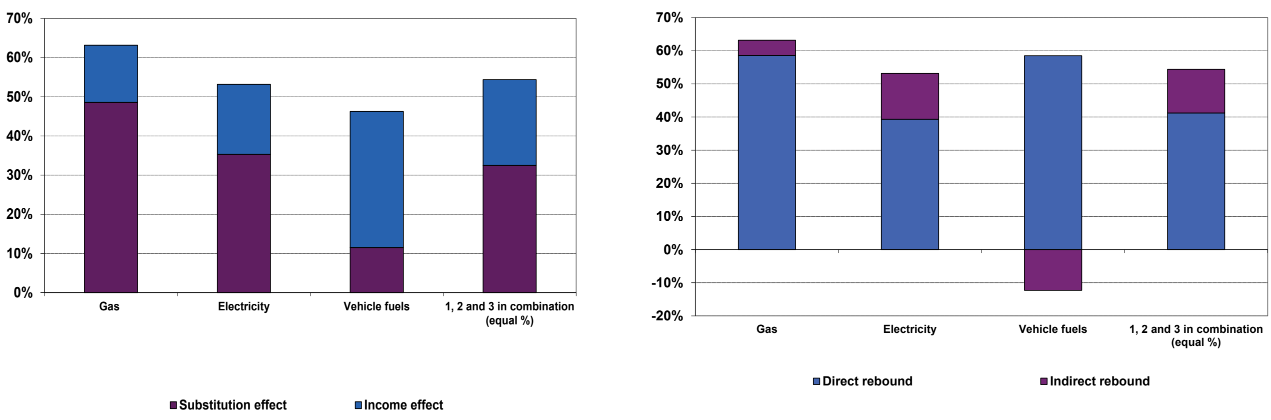

Figure 2. Estimated rebound effects – split by: a) net income and substitution effects; and b) direct and indirect rebound effects.

Table 4. Total expenditure elasticities ($\eta_{q_i x}$).

	Energy			Transport		Food and beverages		Other goods and services				
	Electricity	Gas	Other fuels	Vehicle fuels	Other transport	Food & non-alcoholic beverages	Alcoholic beverages & tobacco	Recreation and culture	Restaurants and hotels	Education	Communication	Other
Expenditure elasticity	0.07	0.15	0.16	1.01	1.33	0.71	0.88	1.22	1.15	1.23	1.06	1.01

Table 5. Total compensated cross price elasticities-energy group ($\tilde{\eta}_{q_i p_j}$).

	Energy			Transport		Food and beverages		Other goods and services				
	Electricity	Gas	Other fuels	Vehicle fuels	Other transport	Food and non-alcoholic beverages	Alcoholic beverages & tobacco	Recreation and culture	Restaurants and hotels	Education	Communication	Other
Electricity	-0.39	0.11	-0.08	-0.04	-0.06	0.04	0.05	0.01	0.01	0.01	0.01	0.01
Gas	0.07	-0.58	0.37	-0.03	-0.04	0.03	0.03	0.01	0.01	0.01	0.01	0.01
Other fuels	0.04	0.12	-0.76	-0.01	-0.02	0.01	0.01	0.003	0.003	0.003	0.003	0.003
Vehicle fuels	0.07	0.15	0.16	-0.55	0.04	0.01	0.01	0.01	0.01	0.01	0.01	0.01

Table 6. Total uncompensated cross price elasticities-energy group ($\eta_{q_i p_j}$).

	Energy			Transport		Food and beverages		Other goods and services				
	Electricity	Gas	Other fuels	Vehicle fuels	Other transport	Food and non-alcoholic beverages	Alcoholic beverages & tobacco	Recreation and culture	Restaurants and hotels	Education	Comms	Other
Electricity	-0.39	0.10	-0.08	-0.06	-0.08	0.02	0.03	-0.01	-0.01	-0.01	-0.01	-0.01
Gas	0.07	-0.59	0.36	-0.05	-0.06	0.02	0.02	-0.01	-0.01	-0.01	-0.01	-0.01
Other fuels	0.04	0.12	-0.76	-0.02	-0.02	0.01	0.01	-0.004	-0.003	-0.003	-0.003	-0.004
Vehicle fuels	0.07	0.15	0.16	-0.59	-0.001	-0.01	-0.01	-0.03	-0.02	-0.02	-0.02	-0.03

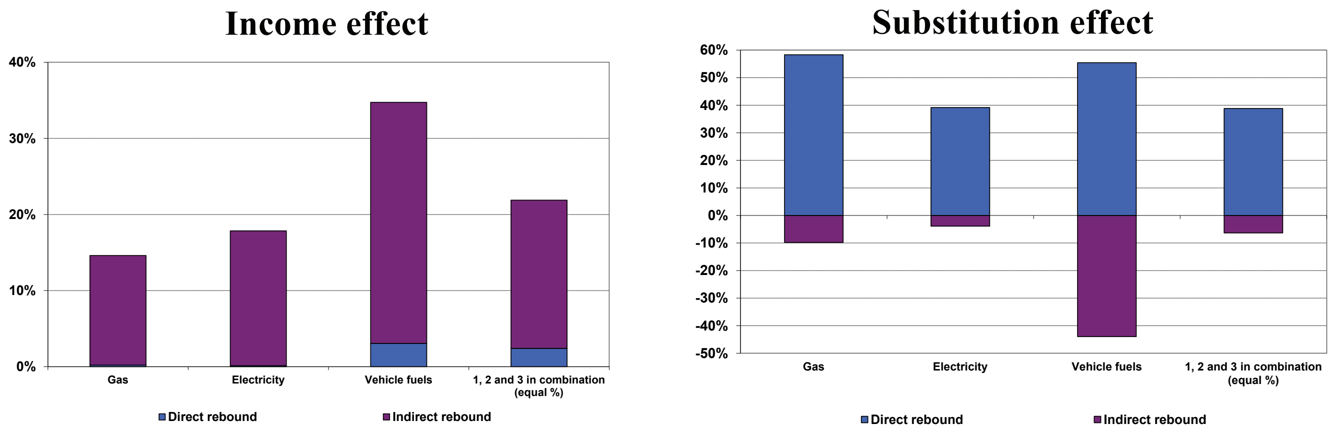

Figure 3. Net income and substitution effects – split by direct and indirect rebound.

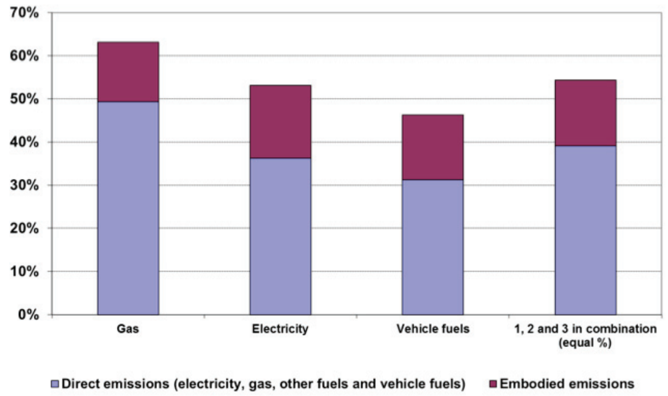

Figure 4. Estimated rebound effects – split by direct and embodied emissions.

Direct emissions from energy commodities account for between two thirds and three quarters of the total rebound (Figure 4). This follows directly from the above, since it is the direct rebound effect that dominates the overall rebound effect. Reduced consumption of other energy commodities slightly reduces the total rebound effect for electricity and gas but has a greater impact on the total rebound for vehicle fuels. Income effects are dominated by embodied emissions (i.e. non-energy commodities) while substitution effects are dominated by direct emissions (i.e. energy commodities) (Figure 5). Since the latter is larger than the former, substitution both within and between energy commodities has the dominant influence on the overall results. Again, studies that neglect substitution effects could erroneously conclude that the total rebound effect consists primarily of embodied emissions – whereas these results show the opposite.

Discussion

In our previous study of combined direct and indirect rebound effects for UK households [8] we concluded that the total rebound effect was modest (0–32 %) for measures affecting electricity and gas and larger (25–65 %) for measures affecting vehicle fuels, and that it primarily derived from increased consumption of non-energy goods and services. We further suggested we may have *underestimated* the total rebound since we did not model substitution effects.

The present study shows that this last suggestion was correct. By using price rather than expenditure elasticities, we now estimate significantly larger rebound effects, namely 63 % for domestic gas use, 53 % for electricity and 46 % for vehicle fuels. The primary source of this rebound is increased consumption of cheaper energy services (i.e. direct rebound), and this is primarily driven by substitution effects. A clear implication of this finding is that studies of combined direct and indirect rebound effects that rely solely upon expenditure elasticities will *underestimate* the total rebound. This is because such studies only capture income effects and not substitution effects. These studies form the bulk of the existing literature since they are easier to conduct.

In practice, many studies focus solely upon *direct* rebound effects and estimate these from time-series data on individual energy services (e.g. transport, heating). Since, by definition, these neglect *indirect* rebound effects, their results may also underestimate the total rebound (unless, that is, the indirect rebound effect is negative). However, our results suggest that such studies may often provide a better approximation to the total rebound effect than do studies that seek to estimate the latter but do so by relying solely upon only expenditure elasticities. Since the direct rebound effect appears larger than the indirect rebound effect, errors in estimating the former will matter more than errors in estimating the latter when used to estimate the total rebound effect.

We suspect, however, that the present study may *overestimate* the total rebound effect. The primary reason for this is that we have modelled efficiency improvements has a reduction in the price of the relevant energy commodities. Hence, we assume the own-price elasticity of energy demand to be equivalent to the efficiency elasticity of energy service demand ($\eta_{q_e,p_e} = \eta_{q_e,\varepsilon}$) and use the former as a measure of the direct rebound effect. As Sorrell and Dimitropoulos [9] show, this equivalence only holds if energy prices are exogenous, energy service demand depends only on energy service prices (p_s) and energy efficien-

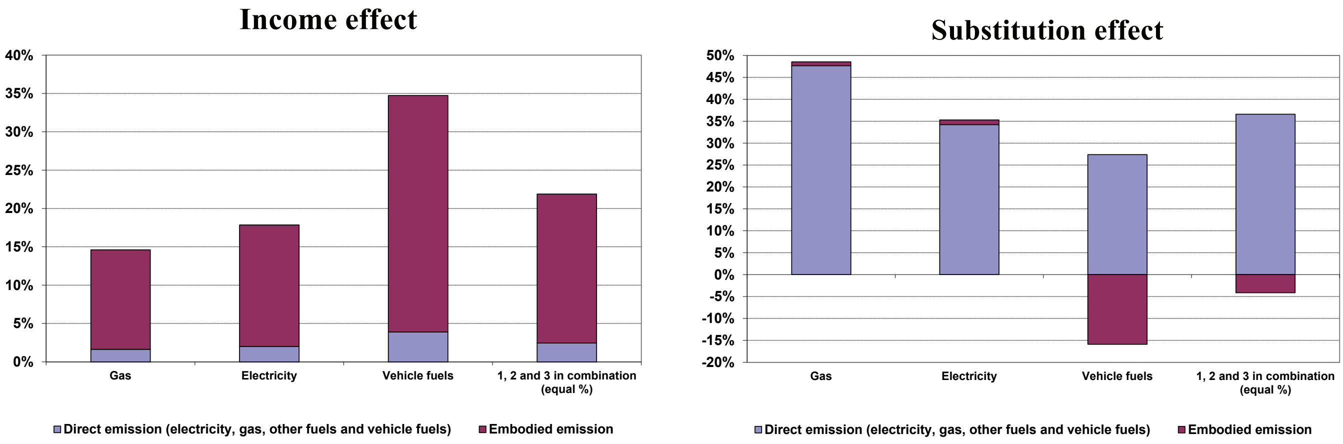

Figure 5. Net income and substitution rebound effects – split by direct and embodied emissions.

cy is constant. Absent these conditions, the own-price elasticity of energy demand will overestimate the direct rebound effect [9]. Factors such as the endogeneity of energy efficiency and the asymmetric response of consumers to changes in energy prices may exacerbate this bias [9]. Hence, the simplicity of using energy commodities rather than energy services in the household demand model comes at a cost.

A further bias may arise when energy commodities provide *multiple* energy services [27]. For example, if electric heating is a complement (substitute) to lighting, the own-price elasticity of electricity may overestimate (underestimate) the direct rebound effect for each. Hunt and Ryan [28] explore this point by estimating a LAIDS model of household energy purchases that includes covariates that they assume to be correlated with energy efficiency. Although not equivalent to including energy services within the demand model, their approach leads to *lower* estimates of energy price elasticities than specifications in which these covariates are omitted. This further suggests that the specification used here may overestimate energy price elasticities and hence also the total rebound effect.

We further observe that our estimates of energy price elasticities are at the high end of the range found in the literature. Lower estimates of these elasticities would lead to lower estimates of the direct rebound effect – and correspondingly higher estimates of the indirect rebound effect. Since energy commodities are relatively GHG intensive, the former is likely to outweigh the latter leading to a lower estimate of the total rebound.

Further caveats relate to the methodological trade-offs discussed – including the limited number of commodity groups employed, the potential sensitivity of the results to separability assumptions and the absence of socio-economic covariates. Our methodology also neglects any supply-side responses to improved energy efficiency which may modify the estimated effects. The likely direction of bias from these sources is ambiguous, although they all represent important avenues for future research. But the priority is to find ways of incorporating energy services directly within a household demand model.

Summary

This study adds to a small but growing volume of evidence that estimates combined direct and indirect rebound effects for households. We extend the existing literature by estimating a full household demand model and identifying the relative contribution of different mechanisms to the results. Our results suggest a total rebound effect of 63 % for measures affecting domestic gas use, 53 % for measures affecting electricity use and 46 % for measures affecting vehicle fuel use. The primary source of this rebound is increased consumption of cheaper energy services (i.e. direct rebound) and this in turn is primarily driven by substitution effects. Our results suggest that previous studies that neglected substitution effects may have underestimated the total rebound effect. However, we have identified a number of reasons why our estimates may be upwardly biased. To reduce this risk, future research should give priority to including energy services directly within a demand model.

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Acknowledgements

This research was supported by funding from the UK Department of Environment Food and Rural Affairs (DEFRA), the UK Economic and Social Research Council (ESRC) and the Scottish Government as part of the Sustainable Lifestyles Research Group (SLRG). Completion of the research was funded by the Research Councils UK through their support for the Centre on Innovation Energy Demand (Grant No. EP/KO11790/1). Earlier versions of this paper were presented at IAEE conferences in New York (June 2014) and Rome (October 2014) and at a BIEE conference in Oxford (September 2014). The usual disclaimers apply.